1. Let $x_1, \ldots, x_n$ be i.i.d. observations of a random variable x.

One sample t-test

- **General statistical assumptions:**

  The observations should follow a normal distribution. The assumption about normality can be relaxed if the data are observations of a continuous random variable with sufficiently large sample size. As a rule of thumb, the one sample t-test is quite reliable if $n > 25$, unless the distribution is very skewed. With sample size $n > 40$, the one sample t-test is quite reliable even for clearly skewed distributions.

- **Null hypothesis:** $H_0 : \mu = \mu_0$ (here, μ denotes the population mean and μ_0 is a fixed value)

  Possible alternative hypotheses:

$$H_1 : \mu > \mu_0 \quad \text{(one tailed)}$$
$$H_1 : \mu < \mu_0 \quad \text{(one tailed)}$$
$$H_1 : \mu \neq \mu_0 \quad \text{(two tailed)}$$

One sample sign test

- **General statistical assumptions:**

  No particular distributional assumptions, except x should at least be ordinal (i.e, the data has a meaningful order or ranking, so that it is possible to determine whether an observation is "greater than", "less than", or "equal to" the hypothesized median).

- **Null hypothesis:** $H_0 : m = m_0$ (here, m denotes the population median and m_0 is a fixed value)

  Possible alternative hypotheses:

$$H_1 : m > m_0 \quad \text{(one tailed)}$$
$$H_1 : m < m_0 \quad \text{(one tailed)}$$
$$H_1 : m \neq m_0 \quad \text{(two tailed)}$$

One sample Wilcoxon signed rank test

- **General statistical assumptions:**

  The test assumes that the distribution of the observations is symmetric about the median under the null hypothesis. The data can be ordinal, but it needs to be interval scaled, i.e., the differences of the data points need to be well-defined for the test statistic to make sense. (For example, there is no intrinsic way to determine the differences of "unhappy", "neutral", or "happy", so this data is not interval scaled.)

- **Null hypothesis:** $H_0 : m = m_0$ (here, m denotes the population median and m_0 is a fixed value)

  Possible alternative hypotheses:

$$H_1 : m > m_0 \quad \text{(one tailed)}$$
$$H_1 : m < m_0 \quad \text{(one tailed)}$$
$$H_1 : m \neq m_0 \quad \text{(two tailed)}$$

Comparison of tests

The sign test requires the mildest distributional assumptions (ordinal data). The Wilcoxon signed test is the second mildest, requiring a symmetry assumption for interval scaled, ordinal data. Finally, the t-test has the strictest assumptions out of the three tests considered (data are observations of a normally distrubuted random variable, although the normality assumption can be relaxed if n is sufficiently large).

The t-test is a statistical test about the population mean, whereas the sign test and the Wilcoxon signed rank test are statistical tests about the population median.