Statistics for Data Science Wintersemester 2023/24 Please complete these problems before the exercise session on Tuesday 5 December, 2023, 8:30. Please be prepared to present your solutions to any problems that you completed successfully.

1. Let x_1, \ldots, x_n be i.i.d. copies of the random variable $x \sim \mathcal{N}(\mu, \sigma^2)$ with $\mu = 10$, $\sigma = 5$, and sample size n = 50.

Generate 100 different random samples x_1, \ldots, x_n using a random number generator. Then, for each sample, compute a 95% confidence interval of the mean. For the simulation of the confidence intervals, assume that the population mean μ and population variance σ^2 used to generate the samples are unknown. How many confidence intervals contain the true population parameter $\mu = 10$ in your experiments?

2. The masses of a certain brand of cookie packages are approximately normally distributed with expected value μ . A set of randomly chosen packages were weighted and the following data (measured in grams) was obtained:

 $398.5,\ 399.9,\ 400.0,\ 398.8,\ 402.8,\ 394.7,\ 398.6,\ 397.5,\ 400.7,\ 401.1.$

- (a) Compute a 95% confidence interval for the mean mass of the cookie packages.
- (b) Compute a 95% confidence interval for the variance of the mass of the cookie packages.
- 3. Download the data file height.txt from the course webpage. The file contains height data for 100 adult males.
 - (a) Calculate the following statistics of the heights: sample mean, standard deviation, variance, minimum, maximum, median, median absolute deviation, mode, skewness, and kurtosis.
 - (b) Visualize the data as a histogram.
 - (c) Compute a 95% confidence interval for the mean height.
- 4. Download the data file FT2.txt from the course webpage. The file contains the durations of 272 consecutive eruptions of the Old Faithful geyser in Yellowstone National Park, Wyoming, USA.

Compute a 95% confidence interval for the mean, variance, and median of the eruption length.